Signals for *CPT* and Lorentz violation in neutral-meson oscillations

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Experimental signals for indirect *CPT* violation in the neutral-meson systems are studied in the context of a general *CPT*- and Lorentz-violating standard-model extension. In this explicit theory, some *CPT* observables depend on the meson momentum and exhibit diurnal variations. The consequences for *CPT* tests vary significantly with the specific experimental scenario. The wide range of possible effects is illustrated for two types of *CPT* experiments presently underway, one involving boosted uncorrelated kaons and the other involving unboosted correlated kaon pairs.

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I. INTRODUCTION

The notion of studying *CPT* symmetry to high precision by using the interferometric sensitivity of neutral-meson oscillations to compare properties of mesons and their antiparticles dates from several decades ago [1]. In recent years, experiments with neutral kaons have constrained the *CPT* figure of merit $r_K \equiv |m_K - m_{\bar{K}}|/m_K$ to about one part in 10^{18}. The most recent published result is $r_K < 1.3\times 10^{-18}$ at the 90% confidence level from the experiment E773 at Fermilab [2]. A preliminary result corresponding to a value of r_K below one part in 10^{18} has been announced by the CPLEAR Collaboration at CERN [3]. Experiments now underway such as KTeV [4] at Fermilab or KLOE [5] at Frascati, as well as other future possibilities [6], are likely to improve these bounds. High-precision *CPT* tests have also been performed with the neutral-*B* system by the OPAL [7] and DELPHI [8] Collaborations at CERN, and other *CPT* measurements in the D, B_d, and B_s systems are likely to be feasible in experiments at the various charm and *B* factories.

On the theoretical front, a purely phenomenological treatment of *CPT* violation in the neutral-kaon system has also existed for decades [1]. It allows for indirect *CPT* violation by adding a complex parameter in the standard expressions relating the physical meson states to the strong-interaction eigenstates.

Although a purely phenomenological treatment of this type is necessary in the absence of a convincing framework for *CPT* violation, it is unsatisfactory from the theoretical perspective. Ideally, *CPT* violation should be studied within a plausible theoretical framework allowing its existence [9]. The purely phenomenological treatment of *CPT* violation can be contrasted with the situation for conventional *CP* violation, where a nonzero value of the phenomenological parameter ϵ_P for T violation can be understood in the context of the usual standard model of particle physics [10–12].

Over the past decade, a promising theoretical possibility for *CPT* violation has been developed. It is based on spontaneous breaking of *CPT* and Lorentz symmetry in an underlying theory [13], perhaps at the Planck scale where one might plausibly expect modifications to the conventional theoretical framework arising from string theory or some other quantum theory of gravity. It appears to be compatible both with experimental constraints and with established quantum field theory, and it leads to a general standard-model extension preserving gauge invariance and renormalizability that can be used as a basis for the phenomenology of *CPT* and Lorentz violation [14,15]. The possibility therefore exists of using *CPT* tests as a quantitative probe of nature at the Planck scale.

An important feature of the standard-model extension is its applicability to a broad class of experiments. It provides a quantitative microscopic framework for *CPT* and Lorentz violation at the level of the standard model, allowing the prediction of signals and the comparison and evaluation of experimental bounds. Investigations related to this theory have to date been conducted for neutral-meson systems [7,8,14,16,17], tests of QED in Penning traps [18–22], photon birefringence and radiative QED effects [15,23–25], hydrogen and antihydrogen spectroscopy [26,27], clock-comparison experiments [28,29], muon properties [30], cosmic-ray and neutrino tests [31], and baryogenesis [32].

The strength of the *CPT* theorem [10] makes it difficult to create a theoretical framework for *CPT* violation that is plausible and avoids radical revisions of established quantum field theory [9,33]. It is therefore unsurprising that in the context of the standard-model extension the phenomenological parameters for *CPT* violation turn out to have properties beyond those normally assumed. Indeed, it has been shown that previously unexpected effects appear, including momentum dependence (in both magnitude and orientation) of the experimental observables for *CPT* violation. The consequences for experimental signals can be substantial and include, for example, the possibility that diurnal variations exist in observable quantities [17].

The present work investigates the theoretical underpinnings of experimental signals for *CPT* violation in the context of the general standard-model extension. One goal is to illustrate how disparate the implications of the momentum and time dependence can be for different experimental scenarios and thereby to encourage the detailed realistic simulations and data analyses required for a satisfactory extraction of *CPT* bounds from any specific experiment.

Some theoretical considerations applicable to all relevant neutral-meson systems are presented in Sec. II, along with some results specific to the kaon system. Experiments on *CP* violation using neutral mesons can be classified according to whether the mesons involved are heavily boosted or not and whether they appear as uncorrelated events or as correlated pairs. The possibility of *CPT*-violating effects dependent on the momentum magnitude or orientation implies substantially different sensitivities to *CPT* violation for these various classes of experiment. In Sec. III, some consequences are developed for two illustrative cases. One is exemplified by the KTeV experiment [4] at Fermilab, in which a collimated beam of highly boosted uncorrelated mesons is studied. The other is exemplified by the KLOE experiment [5] at DAPHNE in Frascati, which involves correlated meson pairs with a wide angular distribution at relatively low boost. The results obtained provide intuition about effects to be expected in various types of experiment, including ones involving other neutral-meson systems.

II. THEORY

This section begins with some theoretical considerations about *CPT* violation and its description in the context of the general standard-model extension, applicable to any of the four relevant types of neutral meson. In what follows, the strong-interaction eigenstates are denoted generically by P^0, where P^0 is one of K^0, D^0, B_d^0, B_s^0. The corresponding opposite-flavor antiparticle is denoted $\overline{P^0}$.

A general neutral-meson state is a linear combination of the Schrödinger wave function for P^0 and $\overline{P^0}$, which can be represented by a two-component object Ψ. The time evolution of the state is determined by an equation of the Schrödinger form [1]:

$$i\partial_t\Psi=\Lambda\Psi, \tag{1}$$

where Λ is a 2×2 effective Hamiltonian. The eigenstates of this Hamiltonian represent the physical propagating states and are generically denoted P_S and P_L. The corresponding eigenvalues are

$$\lambda_S\equiv m_S-\frac{1}{2}i\gamma_S, \quad \lambda_L\equiv m_L-\frac{1}{2}i\gamma_L, \tag{2}$$

where m_S, m_L are the propagating masses and γ_S, γ_L are the associated decay rates. For simplicity, here and in what follows the subscripts P are suppressed on all these quantities and on the components of the effective Hamiltonian Λ.

Flavor oscillations between P^0 and $\overline{P^0}$ are governed by the off-diagonal components of Λ. It can be shown [1] that this system exhibits indirect *CPT* violation [34] if and only if the difference of diagonal elements of Λ is nonzero, $\Lambda_{11}-\Lambda_{22}\neq 0$. The effective Hamiltonian Λ can be written as $\Lambda\equiv M-\frac{1}{2}i\Gamma$, where M and Γ are Hermitian 2×2 matrices called the mass and decay matrices, respectively. The condition for *CPT* violation therefore can in general be written

$$\Delta M-\frac{1}{2}i\Delta\Gamma\neq 0, \tag{3}$$

where $\Delta M\equiv M_{11}-M_{22}$ and $\Delta\Gamma\equiv\Gamma_{11}-\Gamma_{22}$. Note that the elements $M_{11}, M_{22}, \Gamma_{11}, \Gamma_{22}$ are real by definition.

In the context of the general standard-model extension, the dominant *CPT*-violating contributions to Λ can be obtained in perturbation theory, arising as expectation values of interaction terms in the standard-model Hamiltonian [13]. The appropriate states for the expectation values are the wave functions for the P^0 and $\overline{P^0}$ mesons in the absence of *CPT* violation. Since the perturbing Hamiltonian is Hermitian, the leading-order contributions to the diagonal terms of Λ are necessarily real, which means $\Delta\Gamma=0$. The dominant *CPT* signal therefore necessarily resides only in the difference of diagonal elements of the mass matrix *M*. For the general standard-model extension, it follows that the figure of merit

$$r_P\equiv\frac{|m_P-m_{\overline{P}}|}{m_P}\equiv\frac{|\Delta M|}{m_P} \tag{4}$$

provides a complete description of the magnitude of the dominant *CPT*-violating effects. This can be contrasted with the usual phenomenological description, for which the effects of $\Delta\Gamma\neq 0$ should also be considered. For example, contributions involving the diagonal elements of Γ could in principle play an important role.

To make further progress, it is useful to have an explicit expression for ΔM in terms of quantities appearing in the standard-model extension. This has been obtained in Refs. [14,17]. Several factors combine to yield a surprisingly simple expression for ΔM. Since the eigenstates of both the strong interactions and the effective Hamiltonian Λ are eigenstates of the parity operator, and since charge conjugation is violated by the flavor mixing, any *CPT*-violating effects must arise from terms in the standard-model extension that violate *C* while preserving *P*. Also, only contributions linear in the parameters for *CPT* violation are of interest because all such parameters are expected to be minuscule. Moreover, flavor-nondiagonal *CPT*-violating effects can be neglected since they are suppressed relative to flavor-diagonal ones.

The result of the derivation is

$$\Delta M\approx\beta^\mu\Delta a_\mu. \tag{5}$$

In this expression, β^μ is the four-velocity of the meson state in the observer frame: $\beta^\mu=\gamma(1,\boldsymbol{\beta})$. Also, $\Delta a_\mu=r_{q_1}a_\mu^{q_1}-r_{q_2}a_\mu^{q_2}$, where $a_\mu^{q_1}$, $a_\mu^{q_2}$ are *CPT*- and Lorentz-violating coupling constants for the two valence quarks in the P^0 meson, and where the factors r_{q_1} and r_{q_2} allow for quark-binding or other normalization effects [14]. The coupling constants $a_\mu^{q_1}$, $a_\mu^{q_2}$ have dimensions of mass and are associated with terms in the standard-model extension of the form $-a_\mu^q\overline{q}\gamma^\mu q$, where q is a quark field of a specific flavor [35]. It is perhaps worth noting that the flavor-changing experiments on neutral mesons discussed here are the only tests identified to date that are sensitive to the parameters a_μ^q, so bounds from these experiments are of interest independently of any other tests of *CPT* and Lorentz symmetry. Note also that the dependence of ΔM on the meson four-velocity and hence on the meson four-momentum is difficult to anticipate in the context of the usual purely phenomenological description of *CPT* violation, where it seems reasonable *a priori* to take ΔM as independent of momentum. However, the momentum dependence is compatible with the significant changes expected in the conventional picture if the *CPT* theorem is to be violated.

In the next section, a few experimental consequences of the dependence on the meson four-momentum magnitude and orientation are considered, and illustrations of these consequences for specific experiments are given. These examples involve kaons, for which a widely used variable for *CPT* violation is denoted δ_K [1]. It can be introduced through the exact relation between the eigenstates of the strong interaction and those of the effective Hamiltonian:

$$|K_S\rangle = \frac{(1+\epsilon_K+\delta_K)|K^0\rangle + (1-\epsilon_K-\delta_K)|\overline{K^0}\rangle}{\sqrt{2(1+|\epsilon_K+\delta_K|^2)}},$$

$$|K_L\rangle = \frac{(1+\epsilon_K-\delta_K)|K^0\rangle - (1-\epsilon_K+\delta_K)|\overline{K^0}\rangle}{\sqrt{2(1+|\epsilon_K-\delta_K|^2)}}. \qquad (6)$$

In the kaon system, indirect *T* violation is small and any *CPT* violation must also be small. This ensures that δ_K is effectively a phase-independent quantity. However, ϵ_K does vary with the choice of phase convention.

Under the assumption that *CPT* and *T* violation are both small, δ_K can in general be expressed as

$$\delta_K \approx \Delta\Lambda/2\Delta\lambda, \qquad (7)$$

where $\Delta\lambda$ is the difference of the eigenvalues (2) of Λ. In terms of the mass and decay-rate differences $\Delta m \equiv m_L - m_S$ and $\Delta\gamma \equiv \gamma_S - \gamma_L$, one has

$$\Delta\lambda \equiv \lambda_S - \lambda_L = -\Delta m - \frac{1}{2} i \Delta\gamma = -i\frac{\Delta m}{\sin\hat{\phi}} e^{-i\hat{\phi}}. \qquad (8)$$

In this expression, $\hat{\phi} \equiv \tan^{-1}(2\Delta m/\Delta\gamma)$ is sometimes called the superweak angle. Note that a subscript *K* is understood on all the above quantities.

In the context of the standard-model extension, $\Delta\Lambda = \Delta M$ is given by Eq. (5). For a meson with velocity $\boldsymbol{\beta}$ and corresponding boost factor γ, Eqs. (5), (7), and (8) imply

$$\delta_K \approx i \sin\hat{\phi} e^{i\hat{\phi}} \gamma(\Delta a_0 - \boldsymbol{\beta}\cdot\Delta\mathbf{a})/\Delta m. \qquad (9)$$

The figure of merit r_K in Eq. (4) becomes

$$r_K \equiv \frac{|m_K - m_{\bar{K}}|}{m_K} \approx \frac{2\Delta m}{m_K \sin\hat{\phi}} |\delta_K| \approx \frac{|\beta^\mu \Delta a_\mu|}{m_K}. \qquad (10)$$

Using the known experimental values [36] for Δm, m_K, and $\sin\hat{\phi}$ gives

$$r_K \simeq 2\times 10^{-14} |\delta_K| \simeq 2 \left| \beta^\mu \frac{\Delta a_\mu}{1 \ \text{GeV}} \right|. \qquad (11)$$

A bound on $|\delta_K|$ of about 10^{-4} therefore corresponds to a constraint on $|\beta^\mu \Delta a_\mu|$ of about 10^{-18} GeV.

In the above expressions, the explicit momentum dependence arises from the dependence of ΔM on β_μ. Since the eigenfunctions and eigenvalues of Λ depend on M_{11} and M_{22}, the possibility exists that there is also hidden momentum dependence in the parameter ϵ_K, in the masses and decay rates $m_S, m_L, \gamma_S, \gamma_L$, and in the associated quantities Δm, $\Delta\gamma$, $\hat{\phi}$. However, all such dependence is suppressed relative to that explicitly displayed above because the *CPT*-violating contribution to M_{22} is the negative of the contribution to M_{11}. The sole linearly independent source of variation with momentum is therefore the difference ΔM, and only the parameter for *CPT* violation δ_K is sensitive to ΔM at leading order. For example, ϵ_K depends on ΔM at most through correction factors involving the square of the ratio $\Delta M/\Delta\lambda$, which is of order δ_K^2, so for all practical purposes conventional indirect *CP* (*T*) violation displays no momentum dependence in the present framework [37]. The same is true of the other quantities, basically because a small difference between diagonal elements of a matrix changes its eigenvalues only by amounts proportional to the square of that difference.

III. EXPERIMENT

The implications of the four-velocity and hence momentum dependence in the parameters for *CPT* violation can be substantial for experiments with P^0 mesons. The possible effects include ones arising from the dependence on the magnitude of the momentum and ones arising from the variation with orientation of the meson boost [17]. Consequences of the dependence on the momentum magnitude include the momentum dependence of observables, the possibility of increasing the *CPT* reach by changing the boost of the mesons studied, and even the possibility of increasing sensitivity by a restriction to a subset of the data limited to a portion of the meson-momentum spectrum. Consequences of the variation with momentum orientation include a dependence on the direction of the beam for collimated mesons, a dependence on the angular distribution for other situations, and diurnal effects arising from the rotation of the Earth relative to the constant vector $\Delta\mathbf{a}$.

Since in real experiments the momentum and angular dependence are often used experimentally to establish detector properties and systematics, particular care is required to avoid subtracting or averaging away *CPT*-violating effects. However, observation of signals with momentum dependence would represent a striking effect that could help establish the existence of *CPT* violation. It can also suggest new ways of analyzing data to increase the sensitivity of tests. For example, data taken with time stamps can be binned according to sidereal (not solar) time and used to constrain possible time variations of observables as the Earth rotates [17].

The previous section established the momentum dependence of various observables. From the experimental perspective, the expressions obtained can be regarded as defined in the laboratory frame. To display explicitly the time dependence arising from the rotation of the Earth, it is useful to exhibit the relevant expressions in terms of parameters for *CPT* violation expressed in a nonrotating frame. In what follows, the notation and conventions of Ref. [29] are adopted, with the spatial basis in the nonrotating frame denoted $(\hat{X},\hat{Y},\hat{Z})$ and that in the laboratory frame denoted $(\hat{x},\hat{y},\hat{z})$.

In this coordinate choice, the basis $(\hat{X},\hat{Y},\hat{Z})$ for the nonrotating frame is defined in terms of celestial equatorial coordinates [38]. The rotation axis of the Earth defines the $\hat{Z}$ axis, while $\hat{X}$ has declination and right ascension 0° and $\hat{Y}$ has declination 0° and right ascension 90°. This right-handed orthonormal basis is independent of any particular experiment. It can be regarded as constant in time because the Earth's precession can be neglected on the time scale of most experiments, although care might be required in comparing results between experiments performed at times separated by several years or by decades.

In the laboratory frame, the most convenient choice of the $\hat{z}$ axis depends on the experiment but typically is along the beam direction. For example, if the experiment involves a collimated beam of mesons the $\hat{z}$ direction can be taken as the direction of the beam. If instead the experiment involves detecting mesons produced in a symmetric collider, the $\hat{z}$ direction can be taken along the direction of the colliding beams at the intersection point. Since time-varying signals are absent or reduced if $\hat{z}$ is aligned with $\hat{Z}$, in what follows these two unit vectors are taken to be different. Then, $\hat{z}$ precesses about $\hat{Z}$ with the Earth's sidereal frequency Ω, and the angle $\chi\in(0,\pi)$ between the two unit vectors given by $\cos\chi=\hat{z}\cdot\hat{Z}$ is nonzero. For definiteness, choose the origin of time $t=0$ such that $\hat{z}(t=0)$ lies in the first quadrant of the $\hat{X}$-$\hat{Z}$ plane. Also, require $\hat{x}$ to be perpendicular to $\hat{z}$ and to lie in the $\hat{z}$-$\hat{Z}$ plane for all t: $\hat{x}:=\hat{z}\cot\chi-\hat{Z}\csc\chi$. Completing a right-handed orthonormal basis with $\hat{y}:=\hat{z}\times\hat{x}$ means that $\hat{y}$ moves in the plane of the Earth's equator and so is always perpendicular to $\hat{Z}$.

Figure 1 shows the relation between the two sets of basis vectors. For ease of visualization only, the basis $(\hat{x},\hat{y},\hat{z})$ has been translated from the laboratory location to the center of the globe. Note, however, that at the location of the laboratory $\hat{z}$ may lie at a non-normal angle to the Earth's surface. Similarly, the angle χ is unrelated to the colatitude of the experiment unless the $\hat{z}$ axis happens to be normal to the Earth's surface in the laboratory.

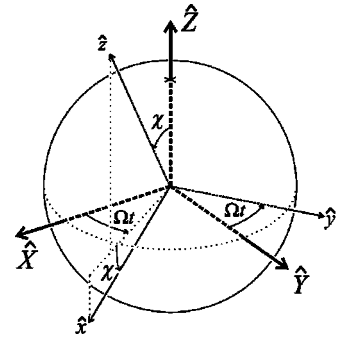

FIG. 1. Bases in the laboratory and nonrotating frames.

Conversion between the two bases can be implemented with a nonrelativistic transformation, given by Eq. (16) of Ref. [29]. This assumes relativistic effects due to the rotation of the Earth can be disregarded, an assumption valid to about one part in 10^6 on the Earth's equator. The time variation of a parameter $\mathbf{a}\equiv(a^1,a^2,a^3)$ for Lorentz violation can then be directly obtained in terms of its nonrotating-frame components (a^X,a^Y,a^Z):

$$a_1(t)=a_X\cos\chi\cos\Omega t+a_Y\cos\chi\sin\Omega t-a_Z\sin\chi,$$
$$a_2(t)=-a_X\sin\Omega t+a_Y\cos\Omega t,$$
$$a_3(t)=a_X\sin\chi\cos\Omega t+a_Y\sin\chi\sin\Omega t+a_Z\cos\chi. \tag{12}$$

The above expressions permit the time variation of the quantity $\boldsymbol{\beta}\cdot\Delta\mathbf{a}$ and hence the time variation of various *CPT* observables to be extracted.

The explicit form of the momentum and time dependence of the parameter δ_K in the kaon system can be found in the general case of a kaon with three-velocity $\boldsymbol{\beta}=\beta(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$ in the laboratory. Here, θ and ϕ are conventional polar coordinates defined in the laboratory frame about the $\hat{z}$ axis. If the $\hat{z}$ axis is the beam axis, these polar coordinates can be identified with the usual polar coordinates for a detector. In terms of these quantities, the above expressions yield

$$\begin{aligned}\delta_K(\mathbf{p},t)=\frac{i\sin\hat{\phi}e^{i\hat{\phi}}}{\Delta m}\gamma(\mathbf{p})\{\Delta a_0+\beta(\mathbf{p})\Delta a_Z(\cos\theta\cos\chi-\sin\theta\cos\phi\sin\chi)\\ +\beta(\mathbf{p})[-\Delta a_X\sin\theta\sin\phi+\Delta a_Y(\cos\theta\sin\chi+\sin\theta\cos\phi\cos\chi)]\sin\Omega t\\ +\beta(\mathbf{p})[\Delta a_X(\cos\theta\sin\chi+\sin\theta\cos\phi\cos\chi)+\Delta a_Y\sin\theta\sin\phi]\cos\Omega t\},\end{aligned} \tag{13}$$

where $\gamma(\mathbf{p})=\sqrt{1+|\mathbf{p}|^2/m_K^2}$ and $\beta(\mathbf{p})=|\mathbf{p}|/m\gamma(\mathbf{p})$, as usual.

The expression (13) is directly relevant for specific experimental and theoretical analyses, including those in the following subsections. One feature of this expression is that the complex phase of δ_K is determined by $i\exp(i\hat{\phi})$, which is independent of momentum and time. The real and imaginary parts of δ_K therefore exhibit the same momentum and time dependence. For instance, Re δ_K and Im δ_K scale together if a meson is boosted. Another feature is that the nature of the *CPT*-violating effects experienced by a meson varies with its boost. For instance, if $\Delta\mathbf{a}=0$ in the laboratory frame then a boosted meson experiences a *CPT*-violating effect greater by the boost factor γ relative to a meson at rest. In contrast, if $\Delta a_0=0$ in the laboratory frame then there is no *CPT*-violating effect for a meson at rest but there can be effects for a boosted meson. The angular dependence in Eq. (13) plays an important role in the latter case. The variation of the size of δ_K according to sidereal time t adds further complications, including the possibility of effects averaging to zero if, as usual, data are taken over extended time periods and no time analysis is performed.

Evidently, the momentum and time dependence displayed in Eq. (13) implies that details of the experimental setup play a crucial role in the analysis of data for *CPT*-violating effects. Next, some issues relevant to two specific and substantially different experiments are discussed. Section III A examines some aspects of an experiment involving collimated uncorrelated kaons with a nontrivial momentum spectrum and high mean boost. Section III B considers a few issues for an experiment producing correlated kaon pairs from ϕ decay in a symmetric collider. The discussions are intended as illustrative rather than comprehensive, since a complete treatment of each case would require detailed simulation of the experimental conditions and lies beyond the scope of this work. The examples provided are chosen to improve intuition about the disparate consequences of the momentum and time dependence of the type (13) in *CPT* observables and to motivate their careful experimental study.

A. Boosted uncorrelated kaons

In this subsection, a few implications of the momentum and time dependence are considered for a particular experimental scenario involving *CPT* tests with boosted uncorrelated kaons. Among possible experiments of this type is the KTeV experiment presently underway at Fermilab [4]. The kaon beam in this experiment is highly collimated and has a momentum spectrum with an average boost factor $\overline{\gamma}$ of order 100, so $\beta\simeq 1$. The geometry is such that $\hat{z}\cdot\hat{Z}=\cos\chi\simeq 0.6$.

In experiments of this type, expression (13) for the momentum and time dependence of δ_K simplifies because the kaon three-velocity reduces to $\boldsymbol{\beta}=(0,0,\beta)$ in the laboratory frame. One finds

$$\delta_K(\mathbf{p},t)=\frac{i\sin\hat{\phi}e^{i\hat{\phi}}}{\Delta m}\gamma[\Delta a_0+\beta\Delta a_Z\cos\chi +\beta\sin\chi(\Delta a_Y\sin\Omega t+\Delta a_X\cos\Omega t)]. \quad (14)$$

All four terms in this expression depend on momentum through the relativistic factor γ. The first two exhibit no time dependence, while the last two oscillate about zero with the Earth's sidereal frequency Ω. Note that a conventional analysis seeking to constrain the magnitude $|\delta_K|$ while disregarding the momentum and time dependence would typically be sensitive to the average value

$$|\overline{\delta_K}|=\frac{\sin\hat{\phi}}{\Delta m}\overline{\gamma}|\Delta a_0+\overline{\beta}\Delta a_Z\cos\chi|, \quad (15)$$

where $\overline{\beta}$ and $\overline{\gamma}$ are the weighted averages of β and γ, respectively, taken over the momentum spectrum of the data.

In many experiments, including KTeV, δ_K is indirectly reconstructed from other observables. It is therefore of interest to identify the momentum and time dependence of the quantities measured experimentally. These include, for example, the mass difference Δm, the K_S lifetime $\tau_S=1/\gamma_S$, and the ratios η_{+-}, η_{00} of amplitudes for 2π decays, defined as usual by

$$\eta_{+-}\equiv\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}\equiv|\eta_{+-}|e^{i\phi_{+-}}\approx\epsilon+\epsilon',$$

$$\eta_{00}\equiv\frac{A(K_L\to\pi^0\pi^0)}{A(K_S\to\pi^0\pi^0)}\equiv|\eta_{00}|e^{i\phi_{00}}\approx\epsilon-2\epsilon'. \quad (16)$$

In the Wu-Yang phase convention [39], it can be shown that $\epsilon\approx\epsilon_K-\delta_K$ [40,41]. Note that $|\epsilon|\simeq 2\times 10^{-3}$ [36] and that $|\epsilon'|\simeq 6\times 10^{-6}$ [42].

Consider first the case where $|\epsilon_K|>|\delta_K|>|\epsilon'|$. This corresponds to the current experimental situation, since $|\delta_K|$ is presently constrained to about 10^{-4}. Neglecting ϵ' then gives

$$|\eta_{+-}|e^{i\phi_{+-}}\approx|\eta_{00}|e^{i\phi_{00}}\approx\epsilon\approx\epsilon_K-\delta_K\approx(|\epsilon_K|+i|\delta_K|)e^{i\hat{\phi}}, \quad (17)$$

where the last expression follows because the phases of ϵ_K and δ_K differ by 90° [43]. Then, it follows that

$$|\eta_{+-}|\approx|\eta_{00}|\approx|\epsilon_K|[1+O(|\delta_K/\epsilon_K|^2)],$$

$$\phi_{+-}\approx\phi_{00}\approx\hat{\phi}+|\delta_K/\epsilon_K|. \quad (18)$$

The above expressions and the results in the previous section show that at leading order in *CPT*-violating parameters the only observable quantities exhibiting leading-order momentum and time dependence are the phases ϕ_{+-} and ϕ_{00}. In terms of experimental observables and parameters for *CPT* violation, one finds

$$\phi_{+-}\approx\phi_{00} \approx\hat{\phi}+\frac{\sin\hat{\phi}}{|\eta_{+-}|\Delta m}\gamma[\Delta a_0+\beta\Delta a_Z\cos\chi +\beta\sin\chi(\Delta a_Y\sin\Omega t+\Delta a_X\cos\Omega t)]. \quad (19)$$

The other experimental observables $|\eta_{+-}|$, $|\eta_{00}|$, ϵ', Δm, $\hat{\phi}$, $\tau_S=1/\gamma_S$ either have no momentum and time dependence or have it suppressed by the square of the parameters for *CPT* violation.

The result (19) shows that an experiment using collimated kaons with a momentum spectrum can in principle place independent bounds on each of the components Δa_0, Δa_X, Δa_Y, Δa_Z. The variation with sidereal time in Eq. (19) is a sum of sine and cosine terms, equivalent to a pure sinusoidal variation of the form $A_{+-}\sin(\Omega t+\alpha)$ with amplitude A_{+-} proportional to $a_\perp\equiv\sqrt{(\Delta a_X)^2+(\Delta a_Y)^2}$ and phase α determined by the ratio $\Delta a_X/\Delta a_Y$:

$$A_{+-}=\beta\gamma\frac{\sin\hat{\phi}\sin\chi}{|\eta_{+-}|\Delta m}a_{\perp}, \quad \tan\alpha=\Delta a_X/\Delta a_Y, \quad (20)$$

Binning in time would in principle allow independent constraints on Δa_X and Δa_Y. A time-averaged analysis would provide a bound on the combination $\Delta a_0+\beta\Delta a_Z\cos\chi$, from which Δa_0 and Δa_Z could be separated if the momentum spectrum is sufficiently broad to permit useful binning in β. In the specific case of the KTeV experiment, it may be feasible to disentangle Δa_X and Δa_Y, but separation of Δa_0 and Δa_Z may be difficult to achieve.

Although the derivation of Eq. (19) neglects the role of ϵ', the small size of the latter ensures that the results remain valid to leading order in *CPT*-violating quantities provided $|\delta_K|$ is not too small. Since the observable *CPT* violation is predicted to occur only in the mixing matrix, ϵ' itself is not directly affected. This restricts its role to momentum- and time-independent corrections to the above equations or to suppressed contributions controlled by the product $\epsilon'\delta_K$. For example, if ϵ' is included one finds Eq. (19) acquires a correction $-\Delta\phi/3$:

$$\phi_{+-}\approx\hat{\phi}-\frac{1}{3}\Delta\phi+\frac{\sin\hat{\phi}}{|\eta_{+-}|\Delta m}\gamma[\Delta a_0+\beta\Delta a_Z\cos\chi + \beta\sin\chi(\Delta a_Y\sin\Omega t+\Delta a_X\cos\Omega t)], \quad (21)$$

where $\Delta\phi\equiv\phi_{00}-\phi_{+-}$. However, this difference contains at most higher-order *CPT*-violating effects [44].

The occurrence of momentum and time dependence in the observables for *CPT* violation has a variety of consequences for the analysis of data in experiments of the type considered in this subsection [17]. These range from direct implications such as distinctive *CPT* signals correlated with the momentum spectrum and time stamps to more indirect consequences such as a *CPT* reach proportional to the boost factor γ under suitable circumstances. There are also some more exotic implications, such as the possibility in principle of improving the *CPT* reach under certain circumstances by examining only a specific momentum range of a subset of the available data. Evidently, a careful experimental analysis allowing for the effects of possible momentum and time dependence has the potential to yield interesting results.

B. Unboosted correlated kaon pairs

In this subsection, some effects of the momentum and time dependence are considered in an experiment testing *CPT* using correlated kaon pairs from ϕ decay in a symmetric collider. The KLOE experiment [5] at DAPHNE in Frascati provides an example of this kind. These experimental circumstances differ substantially from those of the example in the previous subsection, and as a result the *CPT* reach is controlled by different factors. The origin of the kaon pairs in the decay from the ϕ quarkonium state just above threshold implies a line spectrum in the laboratory-frame momentum of about 0.1 GeV for each kaon, so the momentum dependence of the *CPT* observables is relatively uninteresting. In contrast, significant implications for the experiment arise from the wide angular distribution of the kaon momenta in the laboratory frame.

Consider first the general situation of a quarkonium state with $J^{PG}=1^{--}$ decaying at time t in its rest frame into a correlated $P-\overline{P}$ pair. Since the laboratory frame coincides with the quarkonium rest frame, the time t can be identified with the sidereal time introduced earlier. For $P\equiv K$ the relevant quarkonium is the ϕ meson, while for $P\equiv B_d$ it is $\Upsilon(4S)$, for $P\equiv B_s$ it is $\Upsilon(5S)$, and for $P\equiv D$ it is $\psi(3770)$.

Immediately following the strong decay of the quarkonium, the normalized initial state $|i\rangle$ has the form [45]

$$|i\rangle=N[|P_S(+)P_L(-)\rangle-|P_L(+)P_S(-)\rangle], \quad (22)$$

where N is a normalization. In this expression, the eigenstates of the effective Hamiltonian are denoted by $|P_S(\pm)\rangle$ and $|P_L(\pm)\rangle$, where $(+)$ means the particle is moving in a specified direction and $(-)$ means it is moving in the opposite direction.

Suppose at time $t+t_1$ in the quarkonium rest frame the meson moving in the $(+)$ direction decays into $|f_1\rangle$, while the other decays into $|f_2\rangle$ at $t+t_2$. Define for each α the ratio of amplitudes

$$\eta_\alpha\equiv|\eta_\alpha|e^{i\phi_\alpha}=\frac{A(P_L\to f_\alpha)}{A(P_S\to f_\alpha)}. \quad (23)$$

Note that some of these quantities may depend on the momentum and time through a possible dependence on ΔM. Then, the net amplitude $\mathcal{A}_{12}(\mathbf{p},t,t_1,t_2)$ for the correlated double-meson decay into f_1 and f_2 is

$$\mathcal{A}_{12}(\mathbf{p},t,t_1,t_2)=\hat{N}(\eta_2 e^{-i(m_St_1+m_Lt_2)-(\gamma_St_1+\gamma_Lt_2)/2} - \eta_1 e^{-i(m_Lt_1+m_St_2)-(\gamma_Lt_1+\gamma_St_2)/2}), \quad (24)$$

where $\hat{N}=NA(P_S\to f_1)A(P_S\to f_2)$. In this expression, the possible dependence on the three-momenta $\mathbf{p}_1=-\mathbf{p}_2\equiv\mathbf{p}$ of the two mesons and on the sidereal time t has been suppressed in the right-hand side of Eq. (24), but if present it would reside in η_α and $\hat{N}$.

Taking the modulus squared of the amplitude (24) gives the double-decay rate. In terms of $\overline{t}=t_1+t_2$ and $\Delta t=t_2-t_1$, the double-decay rate $R_{12}(\mathbf{p},t,\overline{t},\Delta t)$ is

$$R_{12}(\mathbf{p},t,\overline{t},\Delta t)=|\hat{N}|^2e^{-\overline{\gamma t}/2}[|\eta_1|^2\epsilon^{-\Delta\gamma\Delta t/2}+|\eta_2|^2e^{\Delta\gamma\Delta t/2} -2|\eta_1\eta_2|\cos(\Delta m\Delta t+\Delta\phi)], \quad (25)$$

where $\overline{\gamma}=\gamma_S+\gamma_L$ and $\Delta\phi=\phi_1-\phi_2$.

A detailed study of the *CPT* signals from symmetric-collider experiments with correlated mesons requires simulation with expressions of the type (25) for various final states f_1,f_2. Given sufficient experimental resolution, the dependence of certain decays on the two meson momenta $\mathbf{p}_1,\mathbf{p}_2$ and on the time t could be exhibited experimentally by appropriate data binning and analysis. However, some caution is required because different asymmetries can be sensitive to distinct components of ΔM.

Consider, for example, the case of double-semileptonic decays of correlated kaon pairs in a symmetric collider. Neglecting violations of the $\Delta S = \Delta Q$ rule, for the state $f_+ \equiv l^+ \pi^- \nu$ one finds $\eta_{l^+} \approx 1 - 2\delta_K$, while for $f_- \equiv l^- \pi^+ \overline{\nu}$ one finds $\eta_{l^-} \approx -1 - 2\delta_K$. Inspection of Eq. (25) shows that the double-decay rate $R_{l^+l^-}$ can be regarded as proportional to an expression depending on the ratio

$$\left|\frac{\eta_{l^+}}{\eta_{l^-}}\right| \approx 1 - 2\,\mathrm{Re}\,\delta_K(+) - 2\,\mathrm{Re}\,\delta_K(-)$$
$$= 1 - \frac{4\,\mathrm{Re}(i \sin\hat{\phi} e^{i\hat{\phi}})}{\Delta m}\gamma(\mathbf{p})\Delta a_0. \qquad (26)$$

In the first line of the above expression, the *CPT*-violating contributions from each of the two kaons have been kept distinct because they differ in general for mesons traveling in different directions. All the angular and time dependence in Eq. (13) cancels from the second line because in the present case of a symmetric collider $\boldsymbol{\beta}_1 \cdot \Delta\mathbf{a} = -\boldsymbol{\beta}_2 \cdot \Delta\mathbf{a}$.

The proportionality factor for the double-decay rate $R_{l^+l^-}$ is $|\hat{N}\eta_{l^-}|^2$, which depends on the full expression (13) for δ_K. However, this factor plays the role of a normalization. Unless it is carefully tracked, which could be a potentially challenging experimental task, no angular or time dependence would be manifest in the double-semileptonic decay mode. For instance, the normalization factor would play no role in a conventional analysis to extract the physics using an asymmetry, for which normalizations cancel. Moreover, as mentioned above, the line spectrum in the momentum means that the dependence on $|\mathbf{p}|$ is also unobservable. Indeed, $\gamma(\mathbf{p}) \simeq 1$. The double-semileptonic decay is therefore well suited to placing a clean constraint on the timelike parameter Δa_0 for *CPT* violation, and the experimental data can be collected for analysis without regard to their angular locations in the detector or their sidereal time stamps.

In contrast, certain mixed double-decay modes are sensitive to δ_K only in one of the two decays. This is the case, for instance, for double-decay modes with one semileptonic prong and one double-pion prong. In these situations, the double-decay rate R_{12} in Eq. (25) can be directly sensitive to the angular and time dependence exhibited in Eq. (13), and in particular it can provide sensitivity to the parameters $\Delta\mathbf{a}$ for *CPT* violation. In a conventional analysis, *CPT* violation in a given double-decay mode of this type is inextricably linked with other parameters for *CP* violation [43,46,47]. However, in the present case the possibility of binning for angular and time dependence means that clean tests of *CPT* violation are feasible even for these mixed modes.

Consider, for example, a detector with acceptance independent of the azimuthal angle ϕ. The distribution of mesons from the quarkonium decay is symmetric in ϕ, so the δ_K dependence of a ϕ-averaged data set is governed by the expression

$$\delta_K^{av}(|\mathbf{p}|,\theta,t) \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi\, \delta_K(\mathbf{p},t)$$
$$= \frac{i \sin\hat{\phi} e^{i\hat{\phi}}}{\Delta m}\gamma[\Delta a_0 + \beta\Delta a_Z \cos\chi\cos\theta$$
$$+ \beta\Delta a_Y \sin\chi\cos\theta\sin\Omega t$$
$$+ \beta\Delta a_X \sin\chi\cos\theta\cos\Omega t]. \qquad (27)$$

Inspection shows that by binning in θ and in t an experiment with asymmetric double-decay modes can in principle extract separate bounds on each of the three components of the parameter $\Delta\mathbf{a}$ for *CPT* violation. This result holds independent of other *CP* parameters that may appear, since the latter have neither angular nor time dependence. Combining data from asymmetric double-decay modes and from double-semileptonic modes therefore permits in principle the extraction of independent bounds on each of the four components of Δa_μ.

The same reasoning applies to other experimental observables. For example, one can consider the standard rate asymmetry for K_L semileptonic decays [36],

$$\delta_l \equiv \frac{\Gamma(K_L \to l^+\pi^-\nu) - \Gamma(K_L \to l^-\pi^+\overline{\nu})}{\Gamma(K_L \to l^+\pi^-\nu) + \Gamma(K_L \to l^-\pi^+\overline{\nu})}$$
$$\approx 2\,\mathrm{Re}\,\epsilon_K - 2\,\mathrm{Re}\,\delta_K(\mathbf{p},t), \qquad (28)$$

where the symbol Γ denotes a partial decay rate and $\Delta S = \Delta Q$ has been assumed. In principle, this asymmetry could also be studied for angular and time variation, leading to constraints on Δa_μ.

IV. SUMMARY

This paper has considered some issues involving momentum- and time-dependent experimental signals for indirect *CPT* violation in a neutral-meson system. Effects of this type are predicted in the context of a general standard-model extension allowing for *CPT* and Lorentz violation. Their consequences for data analysis vary substantially with the specifics of a given experiment.

Following a theoretical description of the momentum and time dependence applicable to any neutral-meson system, specific theoretical results are obtained for kaons. Some *CPT* observables depend explicitly on the magnitude and orientation of the meson momentum, which leads to diurnal variations in observables.

Illustrations of the consequences of these results for experiments are provided using two types of scenario already adopted at Fermilab and Frascati, one with collimated boosted uncorrelated kaons and the other with uncollimated correlated kaon pairs from ϕ decay at rest. The implications described for these scenarios also provide intuition about possible effects in other experiments with kaons and in experiments with D or B mesons.

The primary message of this work is that a complete extraction of *CPT* bounds from any experiment allowing for possible momentum dependence and diurnal variation of the observables is a worthwhile undertaking and one that has the potential to yield further surprises from the enigmatic neutral-meson systems.

Note added. The KTeV Collaboration has reported [48] a preliminary bound $A_{+-} \lesssim 0.5°$ on the amplitude (20) of variations of ϕ_{+-} with sidereal time, corresponding to a constraint $a_\perp \lesssim 10^{-20}$ GeV.

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